



UNIVERSITY OF  
TECHNOLOGY SYDNEY

# Quantification and Distribution of Genuinely Nonlocal Resources

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Zhaofeng Su

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Supervisor: Professor Yuan Feng

Centre for Quantum Software and Information

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## CERTIFICATE OF ORIGINAL AUTHORSHIP

I certify that the work in this thesis has not previously been submitted for a degree nor has it been submitted as part of requirements for a degree except as fully acknowledged within the text.

I also certify that the thesis has been written by me. Any help that I have received in my research work and the preparation of the thesis itself has been acknowledged. In addition, I certify that all information sources and literature used are indicated in the thesis.

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## ABSTRACT

Quantum mechanics is nonlocal, which makes it different from classical mechanics. Quantum information and quantum computation is a promising frontier science which is based on quantum mechanics. The nonlocality of quantum mechanics is the fundamental reason that many quantum information processing tasks have significant advantages over their classical counterparts.

In this dissertation, we study three aspects of quantum nonlocality, which include the quantitative relationship between entanglement and nonlocality, the quantification of genuine tripartite nonlocality and the distribution of nonlocal quantum resources. To be specific,

- We investigate the quantitative relationship between entanglement and nonlocality of bipartite quantum systems. We start by exploiting the condition that the nonlocality of two different two-qubit states can be optimally examined by a same nonlocality test setting. Via numerical simulation, we find that the nonlocality of a generic two-qubit state is upper bounded by a function of the corresponding entanglement. Further, we give an analytical proof for the upper bound and find the class of two-qubit states that can reach the upper bound.
- We analyze the quantification of genuine tripartite nonlocality of a generic three-qubit state. We find a method to solve the problem for a special class of three-qubit states which include not only pure states but also mixed states. Applying the method, we drive analytical results for the genuine tripartite nonlocality of generalized GHZ states, generalized Werner states and two class of mixed states with special correlation matrices.
- We investigate the distribution of nonlocal quantum resources. To distribute bipartite nonlocal quantum resources, we design an efficient quantum repeater

scheme, which not only achieves the optimal transmission rate but also consumes less resources of local operations and classical communication, for the case of general pure states of two-qubit system. We also analyze the case of bipartite pure states of arbitrary dimensional quantum system and get an upper bound on the probability of a successful projection operation to produce a maximally nonlocal state. To distribute tripartite nonlocal resources, we propose a simple scenario to generate tripartite nonlocal resources from bipartite nonlocal resources. As examples, we examine the cases that two-qubit Werner states and general two-qubit pure states are prepared as resources, respectively. We get the upper bounds on the genuine tripartite nonlocality that can be generated in the scenario. We also provide the optimal measurement settings to achieve the upper bounds.

**Keywords:** Quantum mechanics; quantum information; local hidden variable model; CHSH inequality; Svetlichny inequality; genuine tripartite nonlocality.